A Quantum Algorithm to Simulate Open Quantum Systems

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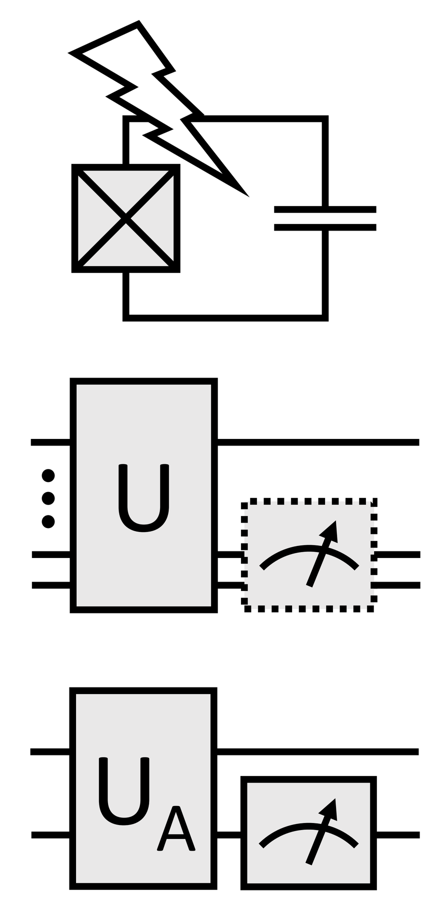
Motivation for Open Quantum Systems (OQS)

Characterization
- No real quantum system is ever truly closed!
- Quantify (non) Markovian environments, tailored error-correction codes

Engineering
- Preparation of entangled or topological states
- Decoherence-free subspaces

Simulation
- OQS as powerful as closed QC
- Non-equilibrium steady states, transport in correlated systems, quantum biology

Prior Work



Analog
- Harness existing open dynamics to emulate OQS (physically non-unitary)

Digital
- Induce open dynamics to create OQS through feedback, measurement, trace-out, etc. (operationally non-unitary)

Parallel
- Reconstruct open dynamics from separate measurements of dilated Kraus operators

- Parallel methods block-encode each Kraus matrix A in its own unitary, which are measured separately
- We must rely on post-selection, and thus fail with some probability
- This probability depends both on the initial state and the embedded operator, neither of which may be known *a priori*
- We propose a quantum algorithm to resolve this drawback

$$U_A = \begin{bmatrix} A & \cdots \\ \vdots & \ddots \end{bmatrix}$$

$$U_A|0\rangle|\psi\rangle = |0\rangle \otimes A|\psi\rangle + |\psi_\perp\rangle$$

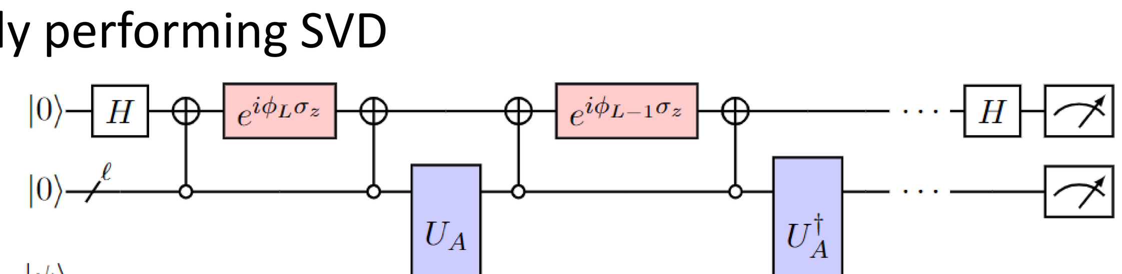
$$\text{if } |0\rangle \to \frac{A}{\sqrt{p_{succ}}}|\psi\rangle$$

$$p_{succ} = \langle\psi|A^\dagger A|\psi\rangle$$

Two-Unitary Decomposition Algorithm (TUD)

- Kraus operators are *contractions*: $\sum_k A_k^\dagger A_k = 1 \rightarrow |A| \leq 1$
- Any contraction A admits a *two-unitary decomposition*[1]: $A = (U_1 + U_2)/2$
- We can use QSVT[2,3] to process a block encoding of A into $\sqrt{1 - A^2}$ without explicitly performing SVD

$$\begin{bmatrix} A & \cdots \\ \vdots & \ddots \end{bmatrix}$$



$$\begin{bmatrix} \sqrt{1 - A^2} & \cdots \\ \vdots & \ddots \end{bmatrix}$$

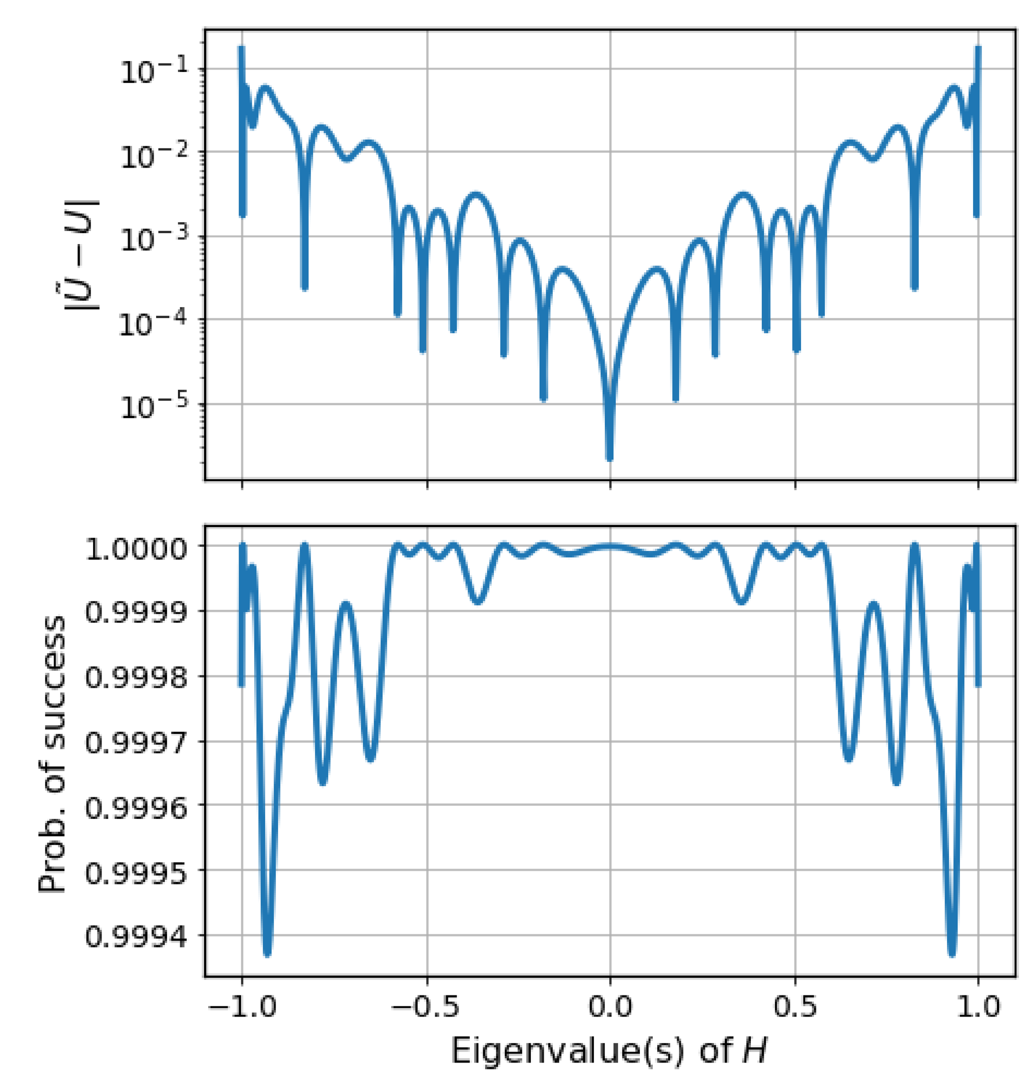
- We use LCU to add/subtract A and i$\sqrt{1 - A^2}$ to produce $U_{1,2}/2$
- With 2 extra calls to the algorithm, oblivious amplitude amplification[5] (OAA) can boost this to $p \approx 1$

Implementation method	State preparation oracle calls	Kraus operator oracle calls
Block	$\mathcal{O}(1/p\ln(1/\beta))$	$\mathcal{O}(1/p\ln(1/\beta))$
TUD	$\mathcal{O}(\ln(1/\beta))$	$\mathcal{O}(1/\delta \ln(1/\epsilon)\ln(1/\beta))$

- We obtain a query complexity independent of the success probability

Error Behavior

- We process $H_1 = (A + A^\dagger)/2$ and $H_2 = i(A - A^\dagger)/2$ to avoid poor error scaling near 0 due to a parity constraint ("four-unitary decomposition")
- With Hermitian inputs, we can rescale and shift the eigenvalues as desired to access low-error portions of the profile (dashed vertical line)



Example: Amplitude Damping Channel

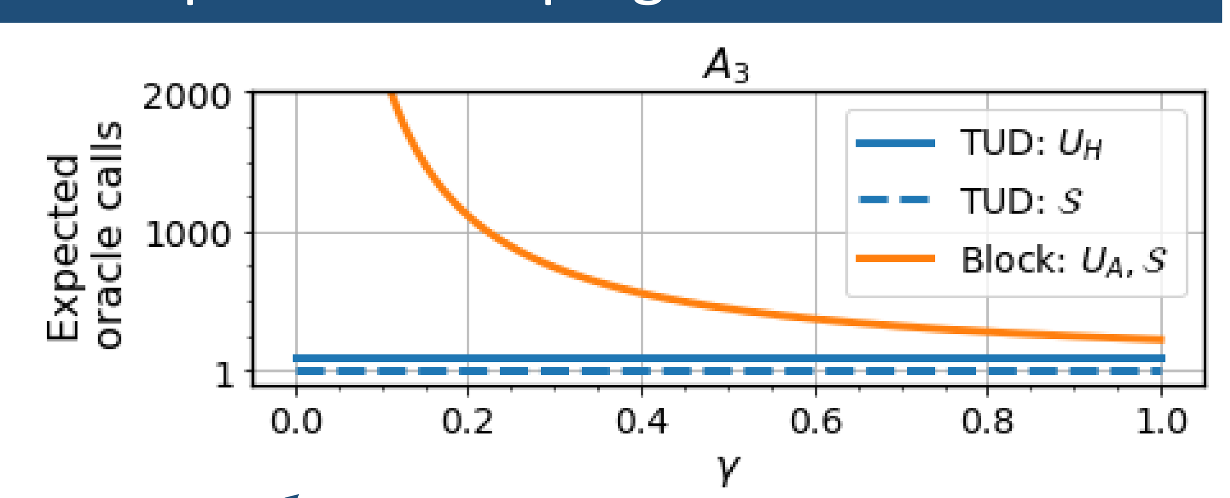
$$\Lambda(\rho) = \sum_k A_k \rho A_k^\dagger$$

$$A_0 = \sqrt{p}\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix}$$

$$A_1 = \sqrt{p}\begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}$$

$$A_2 = \sqrt{1-p}\begin{bmatrix} \sqrt{1-\gamma} & 0 \\ 0 & 1 \end{bmatrix}$$

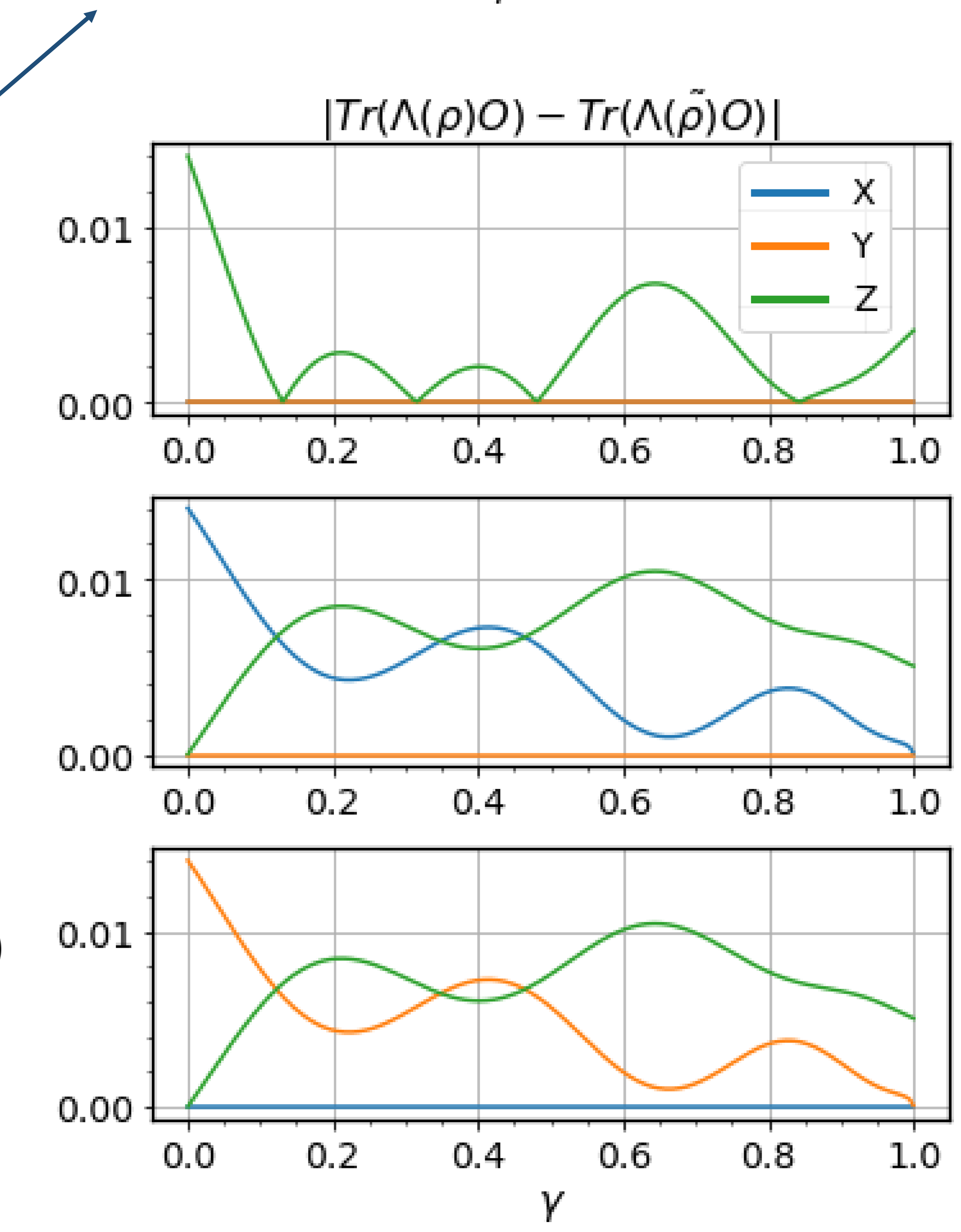
$$A_3 = \sqrt{1-p}\begin{bmatrix} 0 & 0 \\ \sqrt{\gamma} & 0 \end{bmatrix}$$



- We can simulate low-weight Kraus operators with much fewer queries, and this scaling does not depend on the system dimension
- We achieve an error scaling

$$|\langle A_k^\dagger O A_k\rangle - \langle \widehat{A_k^\dagger O A_k}\rangle| \leq 2(\epsilon + h)$$

where ϵ is the error from QSVT and h is the initial block encoding error



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